

# C.U.SHAH UNIVERSITY

## Summer Examination-2019

**Subject Name : Engineering Mathematics - I**

**Subject Code : 4TE01EMT2**

**Branch: B. Tech (All)**

**Semester : 1**

**Date : 14/03/2019**

**Time : 02:30 To 05:30**

**Marks : 70**

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

**Q-1                      Attempt the following questions:                      (14)**

- a) If  $y = \sin 2x \cos 2x$  then  $y_n$  equal to  
 (A)  $\frac{1}{2}(4)^n \cos\left(\frac{n\pi}{2} + 4x\right)$     (B)  $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 4x\right)$   
 (C)  $\frac{1}{2}(4)^n \sin\left(\frac{n\pi}{2} + 2x\right)$     (D) none of these
- b) nth derivative of  $y = \frac{1}{x+a}$  is  
 (A)  $\frac{(-1)^n n!}{(x+a)^{n+1}}$     (B)  $\frac{(-1)^{n-1} n!}{(x+a)^{n+1}}$     (C)  $\frac{(-1)^n n!}{(x+a)^n}$     (D) none of these
- c) If  $1+y = e^{-x}$  or  $y = e^{-x} - 1$ , then  $x$  equal to  
 (A)  $-y + \frac{y^2}{2} - \frac{y^3}{3} + \frac{y^4}{4} + \dots$     (B)  $y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots$   
 (C)  $y + \frac{y^2}{2} + \frac{y^3}{3} + \frac{y^4}{4} + \dots$     (D) None of these
- d) The series  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$  represent expansion of  
 (A)  $\sin x$     (B)  $\cos x$     (C)  $\sinh x$     (D)  $\cosh x$
- e)  $\lim_{x \rightarrow \infty} x^n e^{-ax}$  ( $n$  being a positive integer and  $a > 0$ ) = \_\_\_\_\_  
 (A)  $-1$     (B)  $0$     (C)  $1$     (D) None of these
- f)  $\lim_{x \rightarrow \infty} x \left( a^{\frac{1}{x}} - 1 \right) =$  \_\_\_\_\_  
 (A)  $\log_e a$     (B)  $0$     (C)  $1$     (D) none of these
- g) If  $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)}$  is equal to  
 (A)  $1$     (B)  $-1$     (C) zero    (D) none of these



- h) If  $u = f\left(\frac{x}{y}\right)$  then  
 (A)  $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = 0$  (B)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$  (C)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = u$  (D)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 1$
- i) If  $Q = r \cot \theta$ , then  $\frac{\partial Q}{\partial r}$  is equal to  
 (A)  $\cot \theta$  (B)  $-\cos^2 \theta$  (C)  $\cot \theta - r \operatorname{cosec}^2 \theta$  (D)  $\frac{1}{2} \cot \theta$
- j) If  $u = y^x$ , then  $\frac{\partial u}{\partial x}$  is  
 (A)  $xy^{x-1}$  (B) 0 (C)  $y^x \log x$  (D) none of these
- k) If  $1, \alpha_1, \alpha_2, \dots, \alpha_{n-1}$  are the  $n$  roots of unity, then  $(1 + \alpha_1)(1 + \alpha_2) \dots (1 + \alpha_{n-1})$  is equal to  
 (A)  $n-1$  (B)  $n$  (C)  $-1$  (D) none of these
- l) If  $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$ , then  
 (A)  $a = 2, b = -1$  (B)  $a = 1, b = 0$  (C)  $a = 0, b = 1$  (D)  $a = -1, b = 2$
- m) An  $n \times n$  homogeneous system of equations  $AX = 0$  is given. The rank of  $A$  is  $r < n$ . Then the system has  
 (A)  $n-r$  independent solutions (B)  $r$  independent solutions (C) no solution (D)  $n$  independent solutions
- n) The rank of the diagonal matrix  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$  is  
 (A) 1 (B) 2 (C) 3 (D)  $-2$

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) If  $y = \frac{x^4}{(x-1)(x-2)}$  then find  $y_n$ . (5)

b) Expand  $e^{\sin x}$  as a series of ascending power of  $x$  upto  $x^4$ . (5)

c) If  $u = \log(x^3 + y^3 - x^2y - xy^2)$  then prove that  $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)^2 u = -\frac{4}{(x+y)^2}$  (4)

**Q-3 Attempt all questions (14)**

a) If  $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$  then prove that (5)

$$x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0.$$



b) Prove that  $(1+x)^x = 1+x^2 - \frac{1}{2}x^3 + \frac{5}{6}x^4 - \dots$  (5)

c) Evaluate:  $\lim_{x \rightarrow \frac{\pi}{4}} (1 - \tan x) \sec 2x$  (4)

**Q-4 Attempt all questions** (14)

a) Evaluate:  $\lim_{x \rightarrow 0} \left(\frac{1}{x}\right)^{1-\cos x}$  (5)

b) If  $x = r \sin \theta \cos \phi$ ,  $y = r \sin \theta \sin \phi$ ,  $z = r \cos \theta$  then find  $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$ . (5)

c) Expand  $\log x$  in powers of  $(x-2)$ . (4)

**Q-5 Attempt all questions** (14)

a) If  $u = \sec^{-1} \left(\frac{x^2 + y^2}{x-y}\right)$  then prove that (5)

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\cot u (\cot^2 u + 2).$$

b) Evaluate:  $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - x^2 - 2}{\sin^2 x - x^2}$  (5)

c) If  $y = \cos x \cos 2x \cos 3x$  then find  $y_n$ . (4)

**Q-6 Attempt all questions** (14)

a) The power consumed in an electric resistor is given by  $P = \frac{E^2}{R}$  (in watts). If  $E = 200$  volts and  $R = 8$  ohms, by how much does the power change if E is decreased by 5 volts and R is decreased by 0.20 ohms? (5)

b) Prove that  $\left(\frac{1 + \sin \theta + i \cos \theta}{1 + \sin \theta - i \cos \theta}\right)^n = \cos\left(\frac{n\pi}{2} - n\theta\right) + i \sin\left(\frac{n\pi}{2} - n\theta\right)$  (5)

c) Find the eigen values and eigen vectors of the matrix  $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ . (4)

**Q-7 Attempt all questions** (14)

a) Reduce the matrix  $A = \begin{bmatrix} 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \\ 9 & 10 & 11 & 12 \end{bmatrix}$  to the normal form and find its rank. (5)

b) Find the continued product of all the values of  $\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$ . (5)

c) Prove that  $\sec h^{-1}(\sin \theta) = \log \cot \frac{\theta}{2}$ . (4)

**Q-8 Attempt all questions** (14)

a) Examine whether the following equations are consistent and solve them if they are consistent. (5)

$$2x + 6y + 11 = 0, \quad 6x + 20y - 6z + 3 = 0, \quad 6y - 18z + 1 = 0$$



b) Find the fourth roots of unity and sketch them on the unit circle. **(5)**

c) Verify Caley-Hamilton theorem for the matrix  $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ . **(4)**

